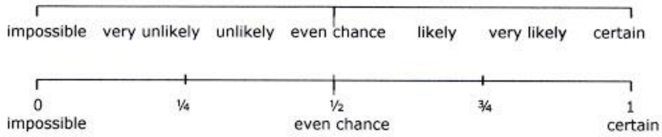
Chapter 6 – Probability

Simple Probability

Probability is a measure of **how likely** an event is to happen.
Probabilities can be written as fractions, decimals or percentages.



An **outcome** is a possible result of an experiment or trial.
Example: when rolling a dice there are 6 different possible outcomes; 1, 2, 3, 4, 5 and 6.

An **event** is a specific thing that has a probability of happening. Example: rolling an even number.

$$P(event) = \frac{Number\ of\ successful\ outcomes}{Total\ number\ of\ outcomes}$$

The probabilities of all outcomes add up to 1.

The **expected frequency** of an event is the number of times you expect the event to happen. This does not mean it will actually happen this many times.
Example: P(Heads on a coin) = ½ so if you flip a coin 10 times you would expect the coin to land on heads 5 times. This may not always happen in real life if you try this but is what should happen in theory.

$$Expected\ Frequency\ of\ Event\ A = P(A) \times number\ of\ trials$$

Experimental Probability

In real-life situations the outcomes of all event aren't equally likely do you have to use results of previous trials to predict future probabilities.
Trial – Each experiment that happens.

$$Estimated\ Probability = \frac{Number\ of\ trials\ with\ successful\ outcomes}{Total\ number\ of\ trials}$$

Estimated Probability is also called Relative Frequency.
The more trials there are, the more accurate the probability should be.
As the number of trials increases, the relative frequency will get closer to the theoretical probability.

Risk

Probability of an event occurring for **negative events**.
Relative frequency can be used to predict bias and assess risk.
For Bias:
A fair coin should land on heads and tails approximately ½ the time each. If the coin is biased it will land on one side more than the other. You can check this by increasing the number of trials and seeing if the P(heads) is getting closer to the theoretical probability of ½.

Risk is when collected data is used to predict how likely a negative event is to happen e.g. a house being flooded or the chance of an 18 year old having a car accident – mostly used by insurance companies to decide how much to charge you.

$$Risk = \frac{Number\ of\ trials\ in\ which\ event\ happens}{Total\ number\ of\ trials}$$

2 types of risk:

1. **Absolute Risk** – how likely an event is to happen. This is just relative frequency.
2. **Relative Risk** – How much more likely an event is to happen for one group compared to another group (e.g. comparing the probability of developing lung cancer for smokers and non-smokers).

$$Relative\ Risk = \frac{Risk\ for\ those\ in\ the\ group}{Risk\ for\ those\ not\ in\ the\ group}$$

Sample Space Diagrams

Sample Space – A **list of all the possible outcomes**.

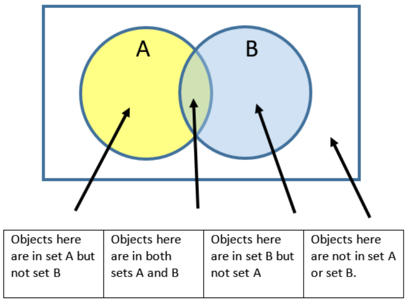
Example: When rolling a fair six-sided dice the sample space is 1, 2, 3, 4, 5, and 6.

Sample Space Diagram – A **table** used to represent the **outcomes of two events**.

	1	2	3	4	5	6
1	1,1	2,1	3,1	4,1	5,1	6,1
2	1,2	2,2	3,2	4,2	5,2	6,2
3	1,3	2,3	3,3	4,3	5,3	6,3
4	1,4	2,4	3,4	4,4	5,4	6,4
5	1,5	2,5	3,5	4,5	5,5	6,5
6	1,6	2,6	3,6	4,6	5,6	6,6

Example: The table on the right shows all the possible outcomes if you roll 2 fair six-sided dice. You can see that there are a total of 36 probabilities.

Venn Diagrams



Uses overlapping circles to represent all the outcomes of two or three events happening.

Each region of a Venn diagram represents a different set of data.

The whole rectangle represents all the possible outcomes.

Venn diagrams can be used to work out probabilities.

Completing Venn Diagrams:

1. Draw and label the Venn diagram
2. Fill in any known values.
3. Use letters to label any area where you don't know the formulae.
4. Work out missing values.
5. The sum of all probabilities in a Venn diagram must equal to 1.

Mutually Exclusive and Exhaustive Events

Mutually Exclusive Events – Events that **CANNOT happen at the same time**.
Example: Getting heads and tails on a coin on the same flip.

For 2 mutually exclusive events, A and B:

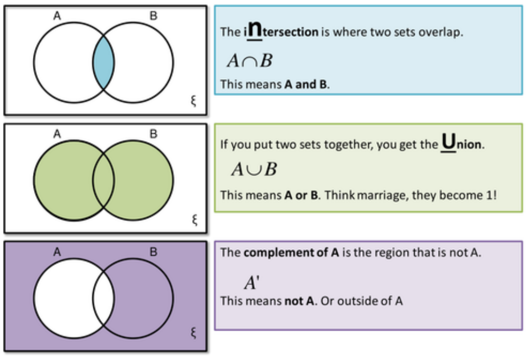
$$\boldsymbol{P(A\ or\ B) = P(A) + P(B)}$$

Exhaustive Events – If the set **contains ALL the possible outcomes**.
Example: When rolling a fair dice, the events P(even) or P(odd) are a pair of exhaustive events as they cover all the possible outcomes you can have when rolling a dice.
The sum of mutually exclusive, exhaustive events is equal to 1.

$$\boldsymbol{P(A) + P(not\ A) = 1}$$
$$\boldsymbol{P(not\ A) = 1 - P(A)}$$

Addition Law

Also known as the **General Addition Law.**
Used for events that are **not mutually exclusive** – events that can happen together.
When two events can happen together and you want to find the probability of both of them happening you don't want to include the overlap – this is the intersection part of the Venn diagram, P(A∩B).



$$\boldsymbol{P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)}$$

Also written as: $\boldsymbol{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$

$P(A \cap B)$ = P(A and B). The intersection/overlap part of the Venn diagram.
$P(A \cup B)$ = P(A or B). On a Venn diagram this is the union of A and B and includes everything in both circles, including the intersection.

Independent Events

Unconnected Events. The outcome of one event does not affect the outcome of the other event.
Example: Flipping a coin and then rolling a dice. The coin landing on tails will not affect what number the dice lands on.

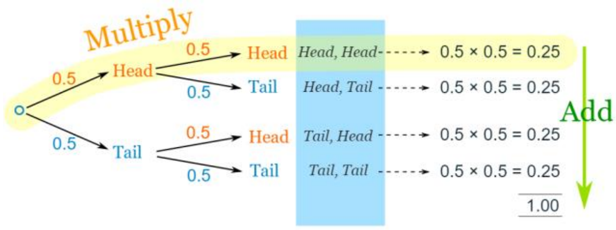
Multiplication Law for Independent Events:

For 2 independent events, A and B: $\boldsymbol{P(A\ and\ B) = p(A) \times P(B)}$

For 3 independent events, A, B and C: $\boldsymbol{P(A\ and\ B\ and\ C) = P(A) \times P(B) \times P(C)}$

$$\boldsymbol{P(at\ least\ 1) = 1 - P(none)}$$

Tree Diagrams



Each branch of a tree diagram represents an outcome.
The probabilities on a set of branches add up to 1.
Always **multiply along the branches** to get the end results.
Add probabilities down columns (after multiplying).
Replacement: The denominator stays the same on the second set of branches. The question will indicate that the item has been replaced or put back
Without replacement: This is not clearly stated in the question. Usually uses words such as 'takes another', 'takes two' – doesn't mention anything about replacing the item. The denominator on the second set of branches will be 1 less than that of the first set.

Conditional Probability

Opposite of independent events.
When one event affects the chances of another event happening.
Example: If there are 2 green and 4 white balls in a bag and you take a white ball the first time and don't put it back, this changes the probability of taking a green or white ball the second time. P(white first time)=4/6, P(white second time) = 1/5, P(green second time) = 4/5. So the chances of selecting a specific colour ball the second time depends on which colour was chosen the first time as choosing white first time increases the chances of selecting green the second time.

Notation:
$P(B|A) = P(B \text{ given that } A \text{ happens})$. The event that happens first comes last in the bracket.

How to know it is conditional probability?
Phrases like **'given that'**, **'if'** or the questions starts by telling you about one group and asks you to work out the probability of a second event **from** '**that**'/'**this**' group.

$$P(B|A) = \frac{P(A \text{ and } B)}{P(B)}$$

$$P(A \text{ and } B) = P(B|A) \times P(A)$$

For two independent events, A and B P(A) = P(A|B).
This formula can be used to test if 2 events are independent. If P(A) and P(A|B) are not equal, the events are not independent but are conditional.